Fracture Technology Associates

10 Questions About ACR

(What you always wanted to know)

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1.0 What is ACR?

The adjusted compliance ratio (ACR) is an experimental method for estimating ΔK_{eff} based on cyclic crack-tip strain. It differs from traditional methodology because it accounts for crack-tip strain below the opening load. It is based on the simple premise that there exists a one to one correspondence between the near crack-tip elastic strain range and ΔK_{eff}. In the absence of crack closure, the relationship between near crack-tip strain and the applied stress or force are linear. In the presence of crack closure, a non-linear response is the result. The ratio of the actual near crack-tip strain range compared to the near crack-tip strain range that would have occurred in the absence of closure is directly proportional to $\Delta K_{eff}/\Delta K_{applied}$. This proportionality is referred to as the compliance ratio [1].

Since near crack-tip strain measurement is impractical, if not impossible, compliance ratio, by itself, has limited practical value. However, the compliance ratio can also be determined for any remote location such as a crack-mouth clip gage or a back-face strain gage. If the initial compliance (δ_i/P) prior to initiation of crack is subtracted, then the resulting compliance is due solely to the presence of the crack. This is referred to as the adjusted compliance ratio (Figure 1). This ratio appears to be independent of the strain or displacement measurement location and can be a good estimate of ΔK_{eff} if proper boundary conditions are met [2].

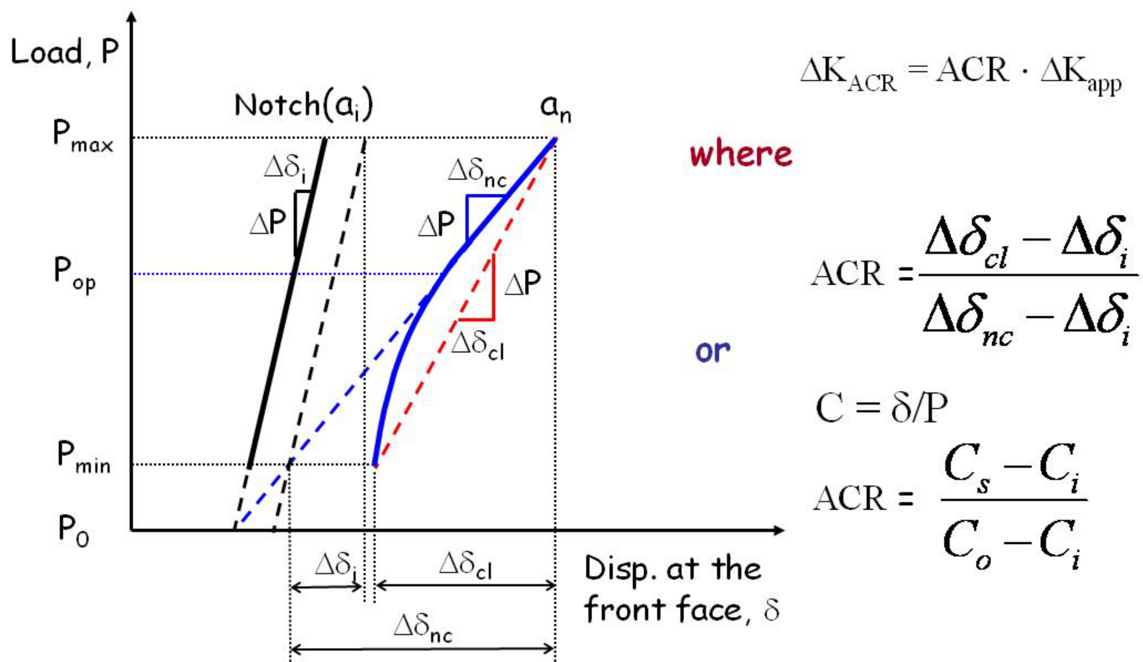


Figure 1 – Illustration of ACR method

2.0 How does ACR differ from the conventional opening load concept?

The conventional opening load concept experimentally determines evidence of contact in the crack wake by measuring an arbitrary deviation in the linearity of the force versus displacement or strain response. Depending on the measurement location, differing opening loads will be inferred from the same arbitrary deviation in linearity. This point of deviation in linearity is evidence of contact in the crack wake, but the location of this contact and its effect on ΔK_{eff} is indeterminate. In principle the opening load method is determining the load at which the crack is fully open, and only the cyclic range above the opening load is used for estimating ΔK_{eff}. In practice the three-dimensional nature of crack face topography including roughness, tunneling, slant fracture surfaces, and surface plasticity are believed to bias opening loads unrealistically high, which in turn biases ΔK_{eff} unrealistically low.

In contrast, the ACR method is one example of a partial closure model that assumes crack tip strain below the opening load, even at P_{min}, and assumes that damage accumulation is proportional to cyclic crack tip strain. The opening load has no direct bearing on the estimation of ΔK_{eff}. The ACR methodology accounts for crack wake forces at P_{min} but is also indeterminate since the exact crack wake force distribution in unknown. However, the ACR method provides a reasonable estimate if certain simplifying assumptions are satisfied.

3.0 What are some unique characteristics of the ACR method?

Experimental and analytical evidence suggests that the ACR method is measurement location insensitive, that is, different measurement locations (near crack-tip or remote) give the same value for ACR. Furthermore, there is no arbitrary offset definition required to arrive at a result. Although examples are numerous in establishing engineering quantities and include yield strength, fracture toughness as well as opening load, ACR is unique in estimating the crack tip cyclic strain and does not rely on the fully-open-crack assumption of the opening load method. The accuracy of the ACR determination is limited by the fidelity of the load-displacement data, not by an arbitrary definition.

In reality, the ACR method is most suitable for removing the effect of remote closure, leaving intact local crack tip closure that is common to both the small crack and long crack. The excellent correlation between the ACR corrected long crack threshold and the physically small crack threshold is shown in Figure 2. Note that the opening load corrected data provide a poor correlation between long crack and small crack behavior [3].

Another example is the work of Zonker, et.al. [10]. An important aspect of Zonker's work is that residual stress effects were partitioned from M(T) and C(T) crack growth rate data, collapsing the curves onto a unique ACR curve that agreed with residual stress free small crack data. Two materials were used: 7075-T7651 and 2324-T39.

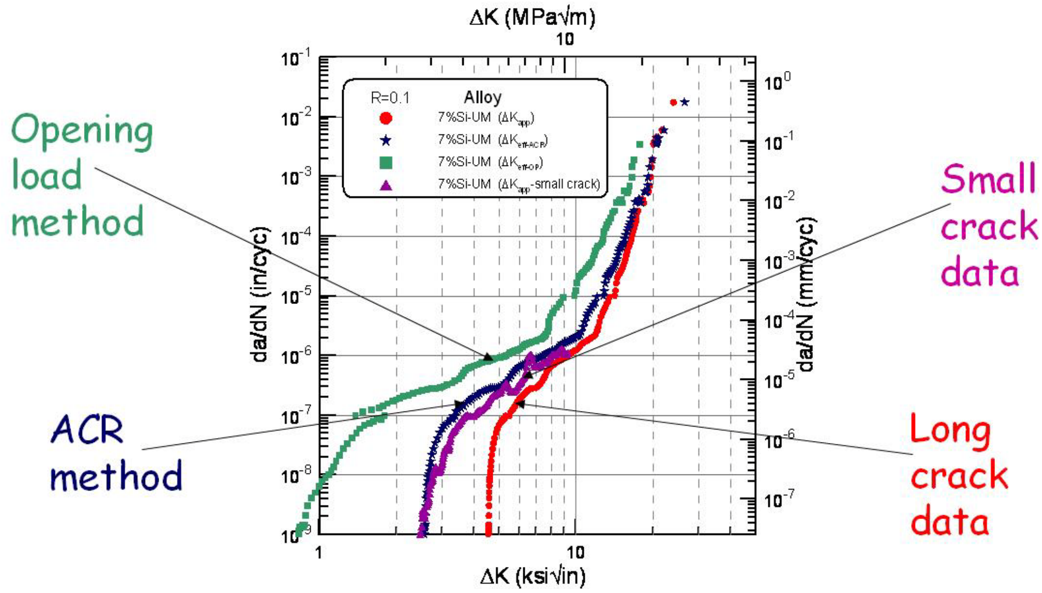


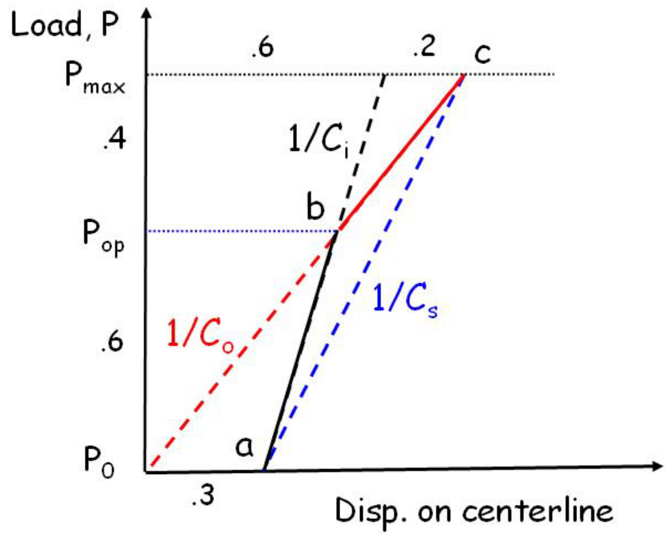
Figure 2 – Correlation between long crack and physically small crack using the ACR method.

4.0 Is there a mathematical model for the ACR method that is an exact solution?

Yes, for the case of a finite crack in an infinite plate, if crack closure is represented by a uniform stress distribution in the crack wake such as that created by hydrostatic pressure, then the ACR method provides an exact solution. This is easy to conceptualize since the solution for the stress intensity and the crack profile are the same for externally applied stress or internally applied stress (pressure). Under externally applied stress, the stress intensity and remote displacement on centerline are easily calculated [4]. In this case, the initial compliance (C_i) is the remote compliance in the absence of a crack. The open compliance (C_o) is the remote compliance in the presence of a crack. In the absence of crack closure, both the open compliance (C_o) and the secant compliance (C_s) increase uniformly and ACR equals one, thus $\Delta K_{eff} = \Delta K_{applied}$.

However, if an internal pressure is applied such that the total of the external and internal stress remain constant, then the crack profile remains the same; the C_s and C_i would be equal to each other; and the value of ACR would be zero. This indicates that ΔK_{eff} would also equal zero, even though an external cyclic stress is being applied.

If the sum of the externally and internally applied stress were to remain constant up to a point (C_i, the closed crack, a $\rightarrow$ b in Figure 3), and then the internal stress remains at zero as the external stress continues to rise (C_o, the open crack, b $\rightarrow$ c in Figure 3), a dual slope load-displacement trace would result. In this simplified case, the value computed from an opening load concept and the ACR method would be identical and both would be correct (Figure 3).



$$\Delta K_{op} = K_{max} - K_{op}$$
$$0.4 = 1.0 - 0.6$$

$$ACR = \frac{C_s - C_i}{C_o - C_i}$$

$$0.4 = \frac{0.5 - 0.3}{0.8 - 0.3}$$

Figure 3 – Illustration of simplifying concept. Both methods give the same result.

However, if the proportionality between the internal and external stress were such that the actual load-displacement trace follows the secant slope (C_s), the ACR would still be correct yet opening load would be undefined. The ACR method does not depend on the shape of the non-linear curve, only the endpoints. Through superposition, this partial closure approach can be solved by first computing the stress intensity at P_{max} from external stress, then computing the stress intensity due to the internal stress at zero load. ΔK_{eff} is the difference between these two.

For this example, P_{min} is assumed to be zero force, but any value of P_{min}, including negative values, will work. It should also be noted that it doesn't matter where the remote measurement is located for these assumptions to be correct. Hence, the ACR method is truly measurement location insensitive.

5.0 But isn't crack tip closure really important?

Not really, but there is a K solution for a concentrated force in the wake of the crack that suggests otherwise [4]. The equation is:

$$K = \frac{\sqrt{2}\,P}{\sqrt{\pi b}}$$

Where P is the concentrated force and b is the distance of this force from the crack tip. The implication is; as b approaches zero, the stress intensity approaches infinity. However, a concentrated force is not realistic. The crack profile adjacent to a concentrated force approaches an asymptote parallel to that force. In reality, concentrated forces do not exist since the point of force is distributed over an area and is limited in magnitude by that area and the yield stress of the material. A more realistic approach is to consider the equation for a uniform pressure up to the crack tip [4] given by:

$$K = \frac{2\sqrt{2}}{\sqrt{\pi}} \rho \sqrt{b}$$

Where ρ is the uniform pressure, and b is the distance over which this stress is applied up to and including the crack tip. In this case, the stress intensity approaches zero as b approaches zero.

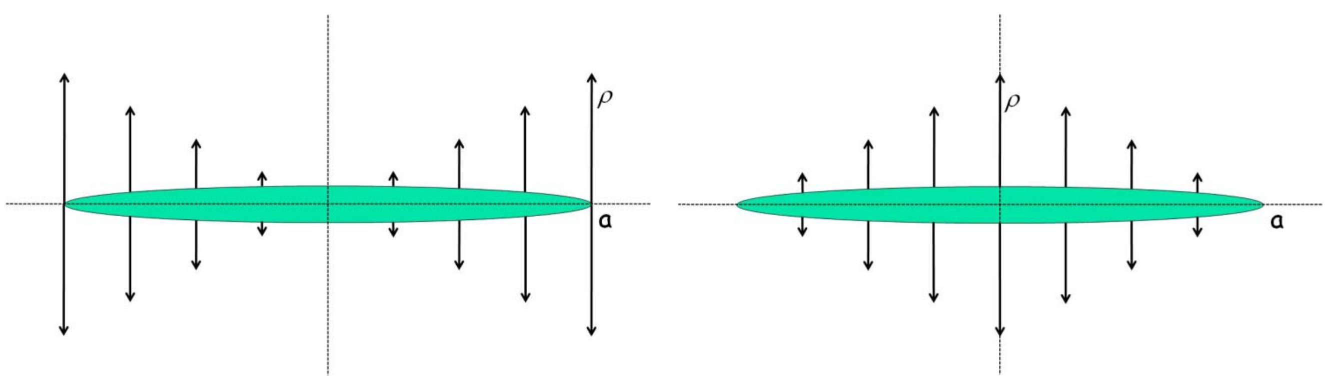
6.0 How close is the ACR model to the real world?

Closed form solutions have been used to show that, for a crack in a finite width plate, the ACR method is exact for an interference location halfway between the notch and the crack tip, assuming that distance is small compared to the original crack length [5]. This observation is based on:

1) Solutions that solve for crack wake force based on remote displacements.
2) Solutions that solve for K based on crack wake force.

Only the force location or profile is needed, not the force magnitude. Numerical integration of these solutions has been used to show reasonable accuracy with realistic crack sizes if a uniform crack wake stress distribution is assumed.

However, if these assumptions about the distribution of crack wake stresses are wrong, these assumptions will lead to errors in estimation of ΔK_{eff}.   For example, if a uniform crack wake stress distribution is assumed and the actual profile varies linearly from zero at the crack mouth to a maximum at the crack tip, then the closure induced K is greater than the assumed value by 27% (Figure 4 – left). If the stress profile were a maximum at the crack mouth and zero at the crack tip, then the closure induced K is less than the assumed value by 38% (Figure 4 - right). The associated errors in ΔK_{eff} would depend on the absolute value of the closure induced K that is then subtracted from K_{max} to give ΔK_{eff}. This analysis also shows the relative insignificance of crack tip closure since the two K solutions in the examples above differ by less than a factor of two, despite having infinitely different internal stresses at the crack tip.



$$K = \frac{2}{\pi} \rho \sqrt{\pi a} \qquad K = (1 - \frac{2}{\pi}) \rho \sqrt{\pi a}$$

Figure 4 – Illustration of differing crack wake stress profiles on stress intensity [4].

For load-shedding decreasing-K, the errors associated the ACR method are assumed to be small. At P_{min} for R ~ 0, the crack is probably closed with crack wake forces applied all the way back to the notch.

The opening load method also has errors in estimating ΔK_{eff}. One source of error is related to the difficulty in establishing the opening load. Another source of error is related to ignoring crack tip strain below the opening load, which can be a significant source of error at high opening loads.

7.0 Are there other partial closure models?

Yes, another intriguing partial closure model assumes the interference in the crack wake is modeled as a thin rigid wedge of uniform height with a small gap at the crack tip [6,7]. This is done by combining the solution for K due to the presence of this wedge with the relationship between K and the crack profile for remote stress. The ratio between the K due to the wedge with no external stress and the K resulting from an applied stress at first point of contact with the wedge is equal to $2/\pi$. This is so regardless of the size of the gap at the crack tip as long as this gap is small compared to the crack size. Experimental evidence has shown that by applying this $2/\pi$ correction to the opening load, the results are surprisingly similar to the ACR method, despite the difference in concept. The common ground is that both ACR and the $2/\pi$ correction methods account for crack tip strain below the opening load. Unfortunately, the $2/\pi$ correction requires that opening load be determined and is therefore subject to the difficulties associated with that inexact measurement.

8.0 What are some limitations of ACR method?

The ACR method assumes the crack closes all the way back to the notch and that force is applied there. Under increasing K, especially in mid Region II or higher, that is probably not the case. Using the initial compliance associated with the notch will result in an over-estimation of ΔK_{eff} if most of the crack wake force is near the crack tip.

9.0 Is there value in measuring CR even if the accuracy of the ΔK_{eff} estimate is in doubt?

Yes, the compliance ratio can be used to compute the displacement or strain due to closure at a particular measurement location. It is not sufficient to compare analytical models with experimental results if only opening load is compared. Crack closure is a complicated process and experimental measurements can only measure the combined effects of plasticity, oxide, roughness, and residual stress. However, if models and experimental data do not share similar opening loads and compliance ratios, then one or more of these contributions to closure are probably being ignored or improperly accounted for in the analytical model.

Data collection should include opening loads associated with several offsets as well as the compliance ratio. It is suggested that, locked in the details of this data, are better solutions for estimating ΔK_{eff} than the initial compliance assumption of the ACR method. For example, if the initial compliance were based on the actual slope at P_{min} instead of the measured slope prior to the initiation of a crack, a more accurate estimation of ΔK_{eff} might be the result. Perhaps the opening loads corresponding to compliance offsets of 1, 2, 4, 8 and 16% might prove useful in that regard.

10.0 How can ACR data be used besides estimating the physically small crack threshold?

ACR corrected data have been used in several ways to estimate threshold and to predict crack growth life. As indicated in Question 3, ACR removes remote closure and leaves local closure. Evidence of this is grounded in the various test results where ACR has been compared to small crack and short crack data. Brockenbrough and Bray [11] used the ACR data as intrinsic ΔK_{eff} curves in their small-crack life prediction model to predict S-N curves. They attributed success of the model to the combination of the ACR data and an explicit K_{max} effect introduced in the model.

ACR corrected data in combination with second order K_{max} effects have been used in another context successfully to provide a unique material property "master curve," as shown in Figure 5 [8]. This is done by first correcting the $\Delta K_{applied}$ data using the ACR method, then normalizing the data according to the following equation:

$$K_{norm} = \Delta K_{ACR}{}^{1-n} \cdot K_{max}{}^{n},$$

where *n* is an empirically derived exponent used to collapse the data in a least squares sense (Schive , XX). The K_{norm} mean curve and the value of *n* define a material intrinsic response.

If residual stress is a significant factor, recent enhancements to this methodology include a "crack compliance" determination of $K_{residual}$ [9]. The normalization equation then becomes:

$$K_{norm} = \Delta K_{ACR}{}^{1-n} \cdot (K_{max} + K_{residual})^{n}$$

Note that this form has an advantage beyond the work of Zonker et. al. [10]. In Zonker's work K_{max} effects were negligible; thus closure correction via ACR was adequate to collapse data to a unique curve. In more recent work K_{max} effects were substantial, and the more recent K_{norm} equation that accounts for $K_{residual}$ was necessary.

Bucci et. al. [12] and Ball [13, 14] used this second form to determine a material intrinsic curve for a forged aluminum alloy with confounding residual stress. Life predictions were performed by constructing design curves with K_{max} effects and closure reintroduced. Newman's closure equations were used to reintroduce closure effects. Then life predictions could proceed as normal using the design curves.

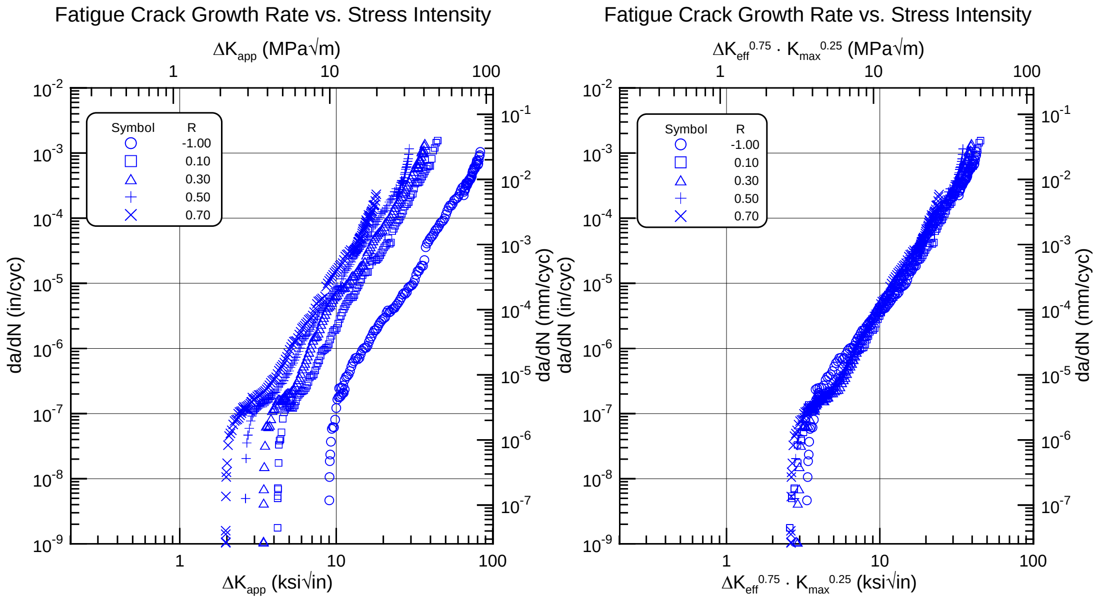


Figure 5 – $\Delta K_{applied}$ data on the left. Normalized "master curve" data on the right.

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